PVP 14

Code: EE4T1

II B.Tech - II Semester – Regular / Supplementary Examinations
October - 2020

COMPLEX VARIABLES & SPECIAL FUNCTIONS
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

PART – A

Answer ***all*** the questions. All questions carry equal marks

11 x 2 = 22M

1.

a) Define analytic function at a point.

b) Find value of *b* such that $u = e^{bx} \cos 3y$ is harmonic.

c) Determine the principle value of $\log(-i)$.

d) Evaluate $\int_C f(z)dz$ where $f(z) = \frac{z+2}{z}$ and C is the semi circle $z = 2e^{i\theta}, 0 \le \theta \le \pi$.

e) Find $\int_0^{1+i} (x^2 - iy)dz$ along the path of y = x.

f) Classify the singularity for the function $f(z) = e^{\frac{1}{z}}$

g) Find the residue of $\frac{ze^z}{(z-1)^3}$ at z = 1.

h) Find the image of $|z| = 2$ under the transformation $w = 3z$.

i) Calculate the fixed points of the of the transformation $w = \frac{6z - 9}{z}$.

j) Prove that $P_2(x) = \frac{3x^2 - 1}{2}$.

k) Find $J_0(2)$.

PART – B

Answer any ***THREE*** questions. All questions carry equal marks.

3 x 16 = 48 M

2. a) Determine the value of p such that the function

$f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}(\frac{px}{y})$ is an analytic function. 8 M

b) If $w = \phi + i\psi$ represents the complex potential function for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$ and determine the function ϕ. 8 M

3. a) Using Cauchy's integral formula, evaluate $\int_C \frac{z - 3}{z^2 + 2z + 5} dz$ where C is the circle $|z + 1 + i| = 1$. 8 M

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region (i) $1 < |z| < 2$ (ii) $|z| > 2$ as Laurent's series. 8 M

4. a) Using Residue theorem, evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where C is $|z| = 3$. 8 M

b)Use the method of contour integration and prove that

$$\int_0^{2\pi} \frac{1}{1+a^2-2a\cos\theta}\,d\theta = \frac{2\pi}{1-a^2}, 0 < a < 1.$$ 8 M

5. a) Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z-plane into a circle in the w-plane. 8 M

b)Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 8 M

6. a)Show that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ 8 M

b) Prove that $P_n(-x) = (-1)^n P_n(x)$ and hence deduce that $P_n(-1) = (-1)^n$. 8 M